A SURVEY OF SOLAR ENVELOPE PROPERTIES USING SOLID MODELLING

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ABSTRACT

Solar envelope is a concept for regulating solar access in urban planning. It is a roof-like imaginary surface over a given piece of land that controls the maximum allowed building height in order to avoid casting shadows on the neighbours during a specific period.

The volume of solar envelopes regulates building density, depending on geometric attributes and time (plot size and proportions, orientation, ground slope, latitude, duration of insolation). This work compares the effect of such factors on the size of solar envelopes on a variety of land parcels, individually or in groups. Repeated applications of solid modelling are used to calculate in each case the values of *'Solar Volume Coefficient'*, i.e. the volume of a solar envelope per unit of its base as a measure for comparisons.

Results show the influence of the various factors affecting the geometry of solar envelopes. Among other findings, it is also shown that solar envelopes generate urban densities lower than conventional urban regulations. The total volume of solar envelopes over an area (*'Solar Building Potential'*) can be increased by raising the reference level of solar envelopes (*'shadow fence'* or *'solar fence'*). Lower urban densities are compensated by facilitating solar applications, as well as by enhancing daylight, ventilation, and vistas in the urban context, thus creating new *'solar cityscapes'* exemplified here on existing street patterns.

KEYWORDS

solar envelope, solar access, solar volume coefficient, solar building potential

1. INTRODUCTION

". . . the solar envelope is a set of volumetric limits that govern building on a particular site. . . . The solar envelope provides the largest developable volume within time constraints. The envelope accomplishes this by defining the largest container of space that would not cast shadows off-site at specified times of the day. We can call these specified times of the day the cut-off times." (Knowles 1981).

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1.1 The meaning of solar envelope

The solar envelope is an urban planning concept related to the availability of direct sunlight on buildings (*'solar access'*) at least during a certain period, in order to ensure the multiple benefits of solar energy—daylight, passive heating, PV energy generation, health matters, plant growth—thus achieving better, more comfortable and sustainable conditions in the built environment. Lepore (2017) presents a historical background of the need for sunlight in urban space since antiquity, showing that it has been an important topic in various eras and regions.

Solar access is impeded by the mutual shading of buildings due to their geometric properties, mainly height. A solar envelope is a hypothetical pyramidal surface based on the boundary of a given site that does not cast shadows on the surroundings during a specified period due to the particular slope of its facets. If a building on the site does not exceed the envelope, then it does not prevent solar rays from reaching its neighbour on the equatorial side. References to orientation in this study assume areas on the northern hemisphere.

The concept has been introduced by Knowles (1981) who championed planning for solar access. Solar envelopes are employed as a legal tool in urban planning to safeguard *'solar rights'* by imposing building height restrictions. Eisenstadt (1982) and Gergacz (1982) are examples of articles on certain legal aspects of such rights in the US.

Such a roof-like form is defined geometrically by the contour of the site and the direction of solar rays at the beginning and end of the minimum period during which solar access to the neighbours should be unobstructed. On the northern hemisphere, those *'cut-off'* moments are usually taken on December 21 (winter solstice) symmetrically to solar noon—say 11:00 and 13:00 to ensure at least 2 hours of direct irradiation. If the solar envelope does not cast shadows at the ends of that period when the sun is at its lowest path, then it will not cast shadows when the sun is higher. To ensure solar access also in summer, similar cut-off limits can be set for summer solstice too, a topic not covered here.

Capeluto & Shaviv (1997) propose two variations of solar envelope: One is *"the 'Solar Rights Envelope' [that] presents the maximum heights of buildings that do not violate the solar rights of any existing buildings, during a given period of the year."* The other is *"the 'Solar Collection Envelope' [that] presents the lowest possible locus of windows and passive solar collectors on the building under consideration, so that they are not shaded by the existing neighbouring buildings, during a given period of winter."* This study focuses on the first type, referring to future buildings.

A solar envelope is a boundary that should not be crossed by any building under it; within the boundary, new buildings can be of any size and form. An alternative approach is proposed by De Luca (2017), aiming to integrate solar access in urban regulations. In that, the form of a new high-rise building is determined not by a pre-set enclosure but by cutting off parts of the proposed volume that block daylight onto existing windows of the neighbouring buildings during a given period. A comparison between this complex computational method and the simple graphic technique proposed by Duncan & Jaffe (1979), both on the topic of solar rights, demonstrates the progress in design for solar access during the last four decades.

1.2 Solar envelope and urban density

Solar envelopes regulate the maximum allowable size of the buildings under them, therefore they control urban density with solar access as the key criterion. Many scholars have studied the correlation of urban fabric with solar access from various perspectives; for instance, energy production (Kantersa & Horvatb, 2012; Shi et al, 2017), daylight availability (De Luca, 2017; Saratsis et al, 2017), solar heating (van Escha et al, 2012; Morgantia et al, 2017). Nevertheless, urban density goes beyond solar access, to additional environmental topics like wind flow, urban heat island, greenery growth, or air quality. Furthermore, it is related to non-environmental issues of urban planning, like land exploitation, property values, population size and distribution, transport systems, etc. This study examines the size of solar envelopes that can be used as a tool for controlling the maximum allowable building volume in relation to topics like the above.

1.3 Solar cityscape

The visual attributes of urban landscape—or *'cityscape'*—depend largely on the building density, which frequently reflects the desire to maximize the built volume over the available land. A common planning instrument to control that, is the maximum allowed building height, often in combination with maximum land coverage and building coefficient. Figure 1 shows a schematic view of a dense urban area, where ideal blocks are vertical prisms of a specified height, separated by an arbitrary network of streets that form *'urban canyons'* due to their width-height proportions. As a result of such geometry, cities suffer from deficits in daylight, ventilation, and free view distance. Additionally, solar applications are generally constrained on roofs since facades are in shade (Stasinopoulos 2002).

Figure 2 shows the same urban example where solar geometry is taken as a key design attribute, specifying a minimum guaranteed duration of direct solar access on all surfaces during the year. The implementation of sun rhythms in urban geometry generates a *'solar cityscape'* radically different than the one of vertical prisms, with obvious advantages in terms of daylight, ventilation, and view, together with the potential of solar applications on building elevations.

1.4 Solar envelope issues

The solar envelope concept in actual applications has to address three issues: (a) its actual usefulness, (b) its ease of use in practice, (c) its effects on building density.

FIGURE 1. A simplified typical cityscape showing the maximum allowable built volume. The urban fabric resembles a vast slab on the ground with cracks (streets) separating vertical prisms (urban blocks). The principal planning restriction is building height. The aim is to have the maximum land use, with little space given to circulation and ventilation between vertical facades that receive limited sunlight and offer restricted views. It is mainly the roofs that have exposure to the sun. North orientation could be at any direction. The street pattern shown here has been taken from the example in 5.2.

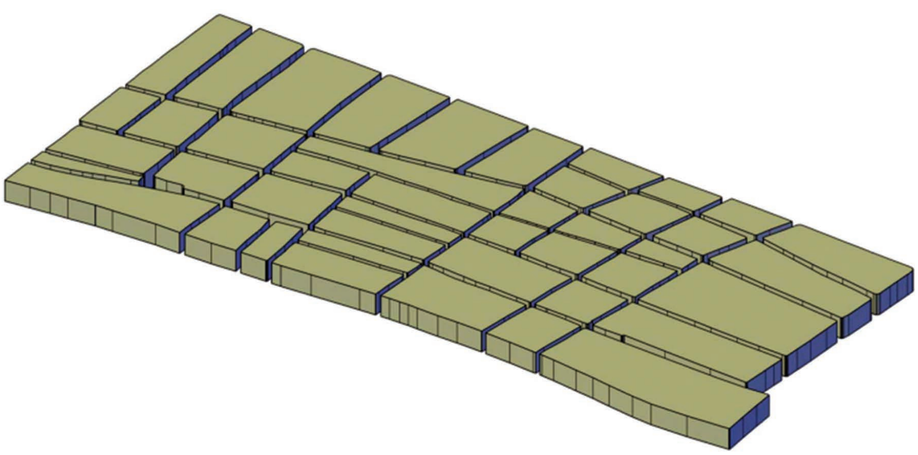
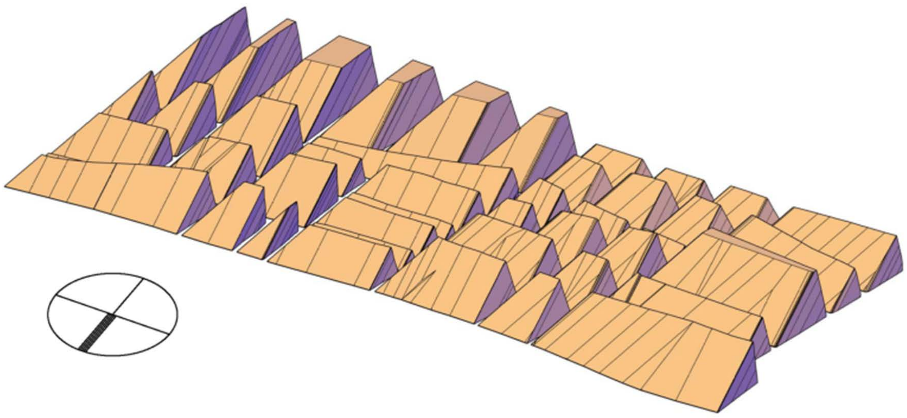


FIGURE 2. A solar cityscape generated with the process described in section 5, with all forms depending on orientation. Resting on the same street layout as in Figure 1, urban blocks are pyramidal islands with most surfaces exposed to solar rays during some period of the day. The maximum allowable height is increased here compared to Figure 1, but that does not have negative effects on daylight and ventilation on lower levels. View from windows is significantly expanded, as well as sky view from the ground level, benefitting city greenery due to more sunlight.



Usefulness

Since the solar envelope aims at maximizing solar irradiation on buildings, it is debateable that it would be beneficial in climates where warm periods prevail to cold ones. Paramita & Koerniawan (2013) advise caution as intense solar availability may aggravate thermal comfort. Knowles admits:

> *"Our need for solar access is not constant but varies according to season. In winter, we value direct access to sunshine for passive heating. In summer, we may actually prefer shade or filtered light to prevent overheating of spaces and building mass. Only for active energy systems, flat-plate collectors for heating water and air or photovoltaics to produce electricity, do we need year-round access to sunshine."* (Knowles 2000 & 2002).

Recognizing the seasonal variations of the benefits from solar radiation in several climatic regions, he distinguishes between winter and summer solar envelopes that aim at solar access and shading respectively, proposing what he calls *'Interstitium'*—i.e. adjustable means between the two envelopes for seasonal amendments (Knowles 2002).

The climatic suitability of solar envelope and its variations is an interesting subject, but beyond the scope of the present work, where the main topic is certain *quantitative* geometric aspects and not the *feasibility* of the concept at a given context.

Ease of use

The geometric generation of a solar envelope on the drawing board is a complex task, usually requiring advanced knowledge of solid geometry for manual construction, or special software and the relevant skills to use it in conjunction with CAD models. That is one of the reasons restraining acceptance of solar envelope in architectural practice.

An answer to that is practical techniques like the solar modelling method described in APPENDIX 1, which is quite straightforward and facilitates multiple applications during the design process for assessing various options easily and fast. For simple applications of geometric nature, the required software is already included in the common CAD programs currently used by most architectural practices. The work presented here has been an actual test of the usability of the method, with repeated applications changing various parameters.

Building density

The volume of a solar envelope determines the maximum size of the building under it, thus being an indirect building coefficient. An argument against the implementation of solar envelope in urban planning arises from the fact that it generally reduces building sizes, especially in the dense central parts of the city where such reductions are undesirable since land values are high. To address such concerns, the volume of the solar envelope and its undesirable effects can be increased by raising the *'solar fence'*, i.e. the lowest level of the desired solar access to the neighbour. In that manner solar availability is ensured to upper floors of buildings, disregarding the bottom ones where direct solar radiation may be of less importance or even undesirable like in shop windows. An alternative method to increase density is by reducing the required minimum duration of solar access, avoiding hours with very low solar altitude that create low solar envelopes.

The proportion between the volume of urban blocks with and without the solar envelope is studied below in part 5, including the option of raising the solar fence. In addition, the study covers the effects of ground slope and orientation on the volume of solar envelope, as indications of the effects of topography on the building density generated by sun-related rules.

2. MODELLING SOLAR ENVELOPES

2.1 Solar envelope construction methods

A solar envelope is used in architectural design as a 3-D boundary that should not be crossed by the building volumes under it. It is a site-specific surface, therefore it has to be created specifically for the given site at an early stage of design to guide subsequent volumetric decisions. The geometric construction of a solar envelope is a complicated task, particularly for complex boundaries or on inclined ground, which used to be resolved with rather laborious descriptive geometry techniques before the computer era and its new tools.

Capeluto & Shaviv (1997) present briefly early work on the topic by various researchers starting from the 1970s, in addition to a model for the design of urban grids and fabric considering solar rights. The same researchers present a computerized version of that model under the name *SustArc* (Capeluto & Shaviv, 2001). Topaloğlu (2003) offers a concise description of the common non-computational methods for the construction of solar envelope, as well as examples of early software: *Solvelope* written in Pascal code by Yeh (1992) and *CalcSolar* in AutoLISP by Kensek (1998). A similar tool was developed for FormZ CAD software by Cotton (1996). Several more tools have been developed in the following years. Freitas et al. (2015) reviewed a wide range of software for solar applications in an urban environment including two for solar envelope, from Morello & Ratti (2005) based on image processing, and from Pereira et al. (2001). Plotnikov (2015) added a *SolarEnvelopeAdvanced* component to Grasshopper/Ladybug collection for Rhino CAD software. The same platform is used by Capeluto & Plotnikov (2017) for a parametric method of constructing such a bounding surface. Canan & Tosunlar

(2016) describe the construction of solar envelope with cutting solar profiles in an educational application.

The above tools handle solar envelope as a 3-D mesh, calculating the position of numerous interconnected nodes. Departing from such complicated approaches, Niemasz et al. (2011) proposed a simple method utilizing Boolean operations in a solid modeller. Vartholomeos (2016) applied such a method for adjusting solar access on residential blocks. Both applications are very similar to a technique presented several years earlier by Stasinopoulos (2000), utilizing normal solid modelling commands of standard CAD software (AutoCAD), as illustrated step-by-step in APPENDIX 1. The following study has been facilitated to a large extend by that *'solar solid modelling'* method, used as an essential tool for examining solar envelope attributes through repetitive applications.

2.2 Use of solar envelope

A basic use of a solar envelope modelled for a site is to check whether a proposed building will allow solar access to the neighbours on its south side during the specified period. By superimposing the envelope produced by solid modelling on a building digital model, any protrusions are immediately visible (Figure 3). The final result can be used as an easily-made proof to authorities or to interested parties on the adherence of a particular design to solar access requirements.

Such small-scale applications can be part of a broader planning scheme, where the solar envelope controls the geometry of urban fabric, primarily the form and size of buildings, hence the urban density and cityscape. In that regulatory role, it is important to know the quantitative range of solar envelopes and the factors that affect them. A survey on that subject is the main topic of this study.

3. SOLAR ENVELOPE AND URBAN DENSITY

3.1 The question

Usual building regulations refer to parameters like building height, free distance between buildings, building coefficient, etc. in *general* terms, without much attention to orientation and topography. In contrast, the restrictions imposed by solar envelopes are considerably affected by varying, site-specific factors like ground slope and orientation that can generate different urban

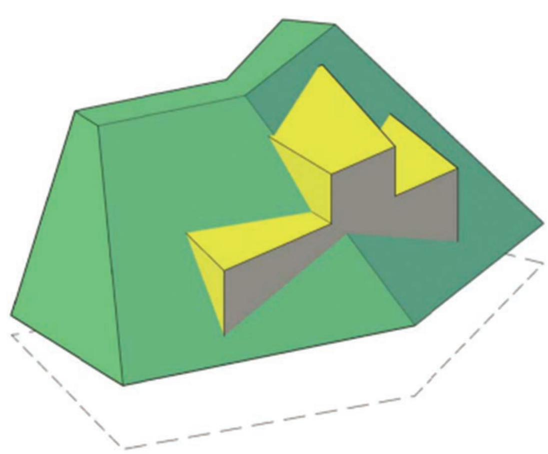


FIGURE 3. Part of the building on the plot exceeds the solar envelope, casting shadows onto the neighbours, thus 'violating their solar rights'. The architect should rearrange the excessive parts elsewhere under the envelope.

volumes on the same 2D layout. The present study explores the effects of such factors on the size of solar envelopes and therefore on the urban density of areas where the concept is applied.

More specifically, the shape and volume of a solar envelope over a specific land parcel depends on factors related to

1. the given plot (size, proportions, orientation),
2. the terrain (ground slope and its orientation),
3. time (duration of solar access that determines the solar angles depending on latitude).

The main research subject of this work is the effect of these factors on the size of the solar envelope, a question investigated via repeated applications of solar solid modelling. The aim is to assess the maximum buildable volume over plots of various attributes as specified by the corresponding solar envelopes.

3.2 'Solar Volume Coefficient': an urban density gauge

In order to make comparison between different plot sizes, the calculated volume of each envelope is divided by its base area, the ratio being the solar envelope volume per square unit of base—called here *'Solar Volume Coefficient'*. Its value corresponds to the average height of the solar envelope over a given plot. In that, it is similar to the *'average height'* of solar volume for unbuilt areas calculated by the SustArc model of Capeluto & Shaviv (2001), which correlates to urban density assuming arbitrary floor heights.

Solar Volume Coefficient can be considered as a kind of building coefficient indicating the volume of a building that is allowed to be built on a land parcel observing the solar rights of the neighbours. In this study it is used as the main quantitative tool for assessing the numerous examined cases.

3.3 Methodology

The work includes two parts: The first examines the volume of solar envelope on *individual land parcels* of various proportions, orientations, and ground slopes. The second focuses on a large *group of urban blocks* on a real street pattern, again assuming different orientation and ground slope values.

In both parts, solar envelopes are constructed assuming a cut-off time on 21 December with solar angles in Izmir, Turkey (latitude 38.4 N, longitude 27.1 E). The individual plots are studied for three lengths of solar aperture, 1, 2 and 4 hours (Figure 4). Similarly, ground is taken at three slopes, 0%, 10%, 20%, with the latter two taken as north- and south-facing, producing totally 11 combinations for each plot. The effect of latitude was tested by repeating the process for the same plot types in Aarhus, Denmark (56.2 N, 10.2 E) but only for horizontal plots (3 combinations). The same assumptions were applied for the urban blocks, with the exception of solar aperture that, for economy, is taken only for 2 hours, thus giving 5 combinations of study. Table 1 summarises the factors examined in each of the two parts. The variables were selected randomly, as experimental assumptions in the search of trends of solar envelope sizes.

4. INDIVIDUAL PLOTS COMPARISON

4.1 The plots of the study

Solar envelopes are generated here for one-hectare (~2.5 acres) rectangular plots with three types of side ratio, 100x100, 71x141, 50x200 meters (1/1, 1/2, 1/4), in order to explore the effect

FIGURE 4. Solar vectors for 1, 2, and 4 hours of insolation, symmetrical to solar noon of 21 December in Izmir. The vectors show the direction of solar rays at the end moments of insolation duration. Solar angles are from www.suncalc.org/#/38.4068,27.1151,15.

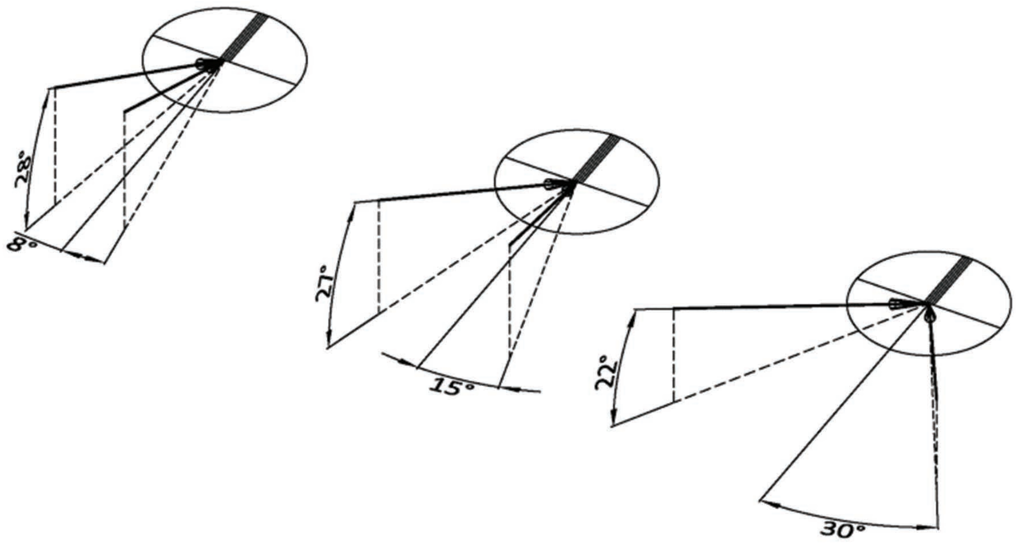


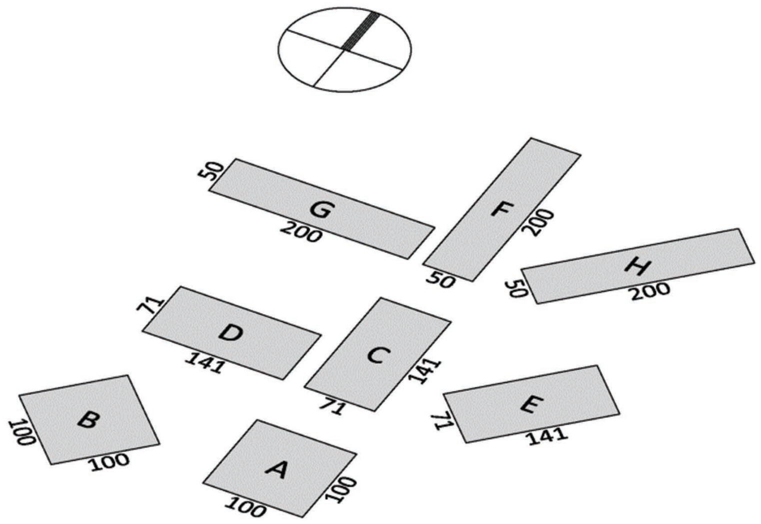
TABLE 1. Factors tested for their effect on the volume of solar envelope.

	Individual plots	Urban blocks
Plan proportions	X	
Orientation	X	X
Ground slope	X	X
Solar aperture	X	X
Latitude	X	
Solar fence		X
Plan size	X	

of proportions of a plot on the volume of its solar envelope. Of course, urban blocks can be larger or smaller than one hectare and can have many shapes other than rectangular. The above geometries are taken only for *experimental reference*, without any link to a particular setting.

Each of the three block types is considered in 2 or 3 orientations, rotated by 0, 45 and 90 degrees as in Figure 5. The envelopes are constructed for each plot for 3 different lengths of solar exposure (1, 2, 4 hours), as shown in Figure 6 to Figure 8, for horizontal ground without height restrictions. The procedure is repeated for slopes 10% and 20%, south and north-facing (axonometric views not shown here).

FIGURE 5. The individual plots of the study. Starting from a square plan of 100x100m (cases A, B), dimension proportions change to 1/2 (C, D, E), and 1/4 (F, G, H) keeping the same total area of one hectare. Orientations differ by 45° and 90°.



After the geometric construction of each envelope using solid modelling as described in APPENDIX 1, its volume is calculated and divided by the base area to find the Solar Volume Coefficient of the particular case. The coefficient of plot A on horizontal ground for one hour of insolation is used as a universal measure of comparison between all cases shown in the graphs of Figure 9 to Figure 15.

FIGURE 6. Solar envelopes for 1 hour of insolation (solar azimuth 8°, altitude 28°). The volume of case A is taken as reference for relative comparison. Numerical output is given further below.

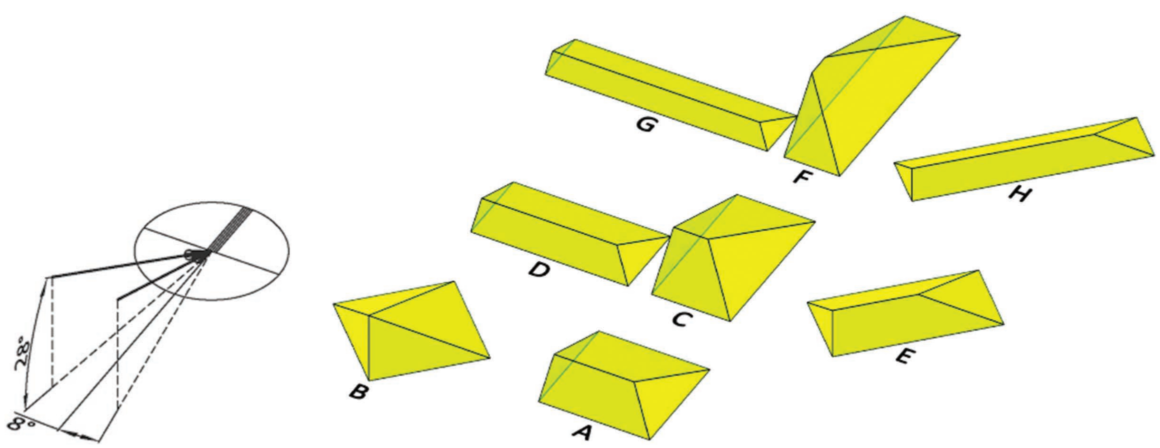


FIGURE 7. Solar envelopes for 2 hours of insolation (solar azimuth 15°, altitude 27°).

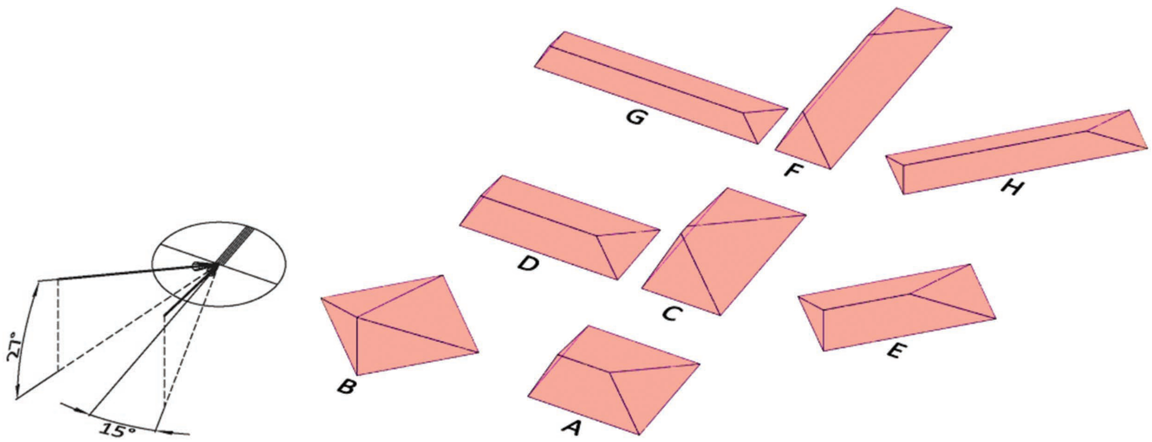


FIGURE 8. Solar envelopes for 4 hours of insolation (solar azimuth 30°, altitude 22°).

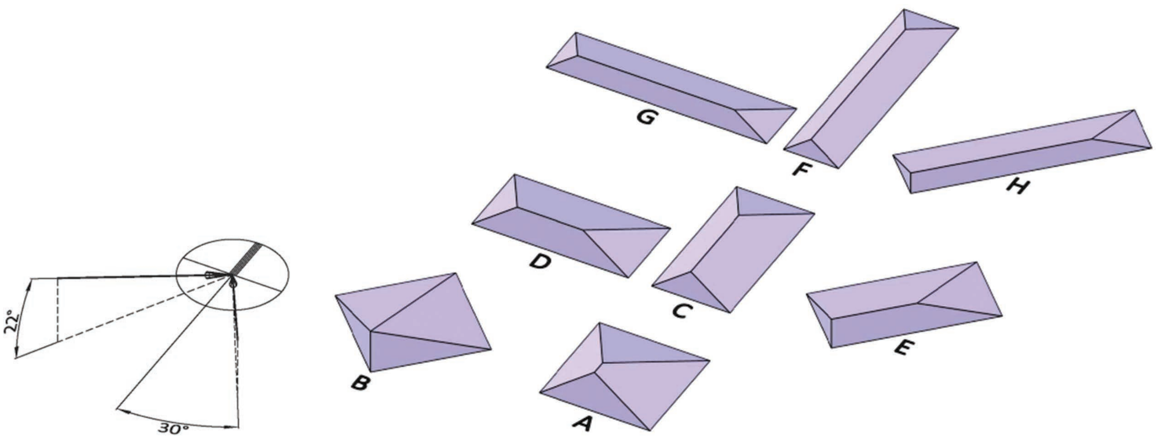
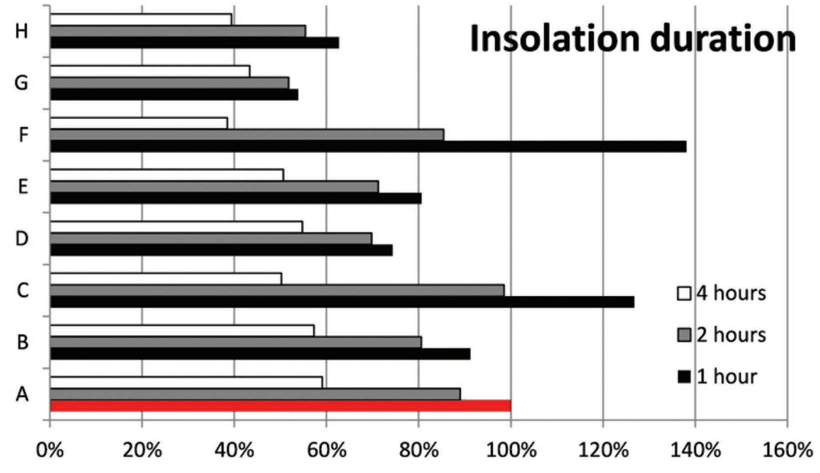


FIGURE 9. Effects of insolation duration on Solar Volume Coefficient as percentage of the reference case A, shown in red at the bottom.



4.2 Effects of plan proportions & orientation

Figure 9 shows that the Solar Volume Coefficient of a plot varies with orientation. The difference is less in square plots (A) and more in elongated plans (C, F) when they are rotated by 45 and 90 degrees, especially for 1 hour of insolation.

Comparing the values for blocks C & D as well as F & G in Figure 9, it appears that, for 1 or 2 hours of exposure, the N-S block orientation (cases C, F) provides larger solar envelope than E-W. The same applies also for the long blocks rotated by 45° (E, H). The E-W orientation (D, G) is more beneficial than the others only for longer exposure times (4 hours). This is due to the taller volumes generated by high solar altitudes on the N-S blocks, an advantage that diminishes for lower angles further from noon.

This observation partially confirms a remark by Knowles (2003) about the effects of orientation on the volume of solar envelopes:

> *"Street orientation: three different block orientations demonstrate the effect on size and shape of solar envelopes; solar envelopes over E–W blocks have the most volume and the highest ridge, generally located near the south boundary (top); N–S blocks produce less volume and a lower ridge running length-wise (middle); diagonal blocks produce the least volume and a ridge along the south-east boundary (bottom)"* (Figure 10).

Figure 9 indicates that Knowles' remark apparently applies only for low sun paths, i.e. for 4 hours exposure time. The same difference is observed in higher latitude too (see 4.4).

4.3 Effects of solar aperture

As shown in Figure 9, the increase of insolation period from 1 to 2 to 4 hours appears to cause a substantial reduction of Solar Volume Coefficient in plots elongated along N-S axis (C, F). This is more intense for 4 hours of insolation due to low sun angles from the sides that decrease the high-peak volumes available for shorter periods. In that case, E-W orientation is more beneficial, confirming Knowles's remark.

4.4 Effects of latitude

To check the effects of latitude, the solar envelopes of plots A-H were constructed also for Aarhus, Denmark (latitude 56.2 N, longitude 10.2 E) (Figure 11). As previously, the Solar

FIGURE 10. According to Knowles (2003), option A has the largest solar envelope and C the smallest. According to the present research, that seems to apply only for low sun angles.

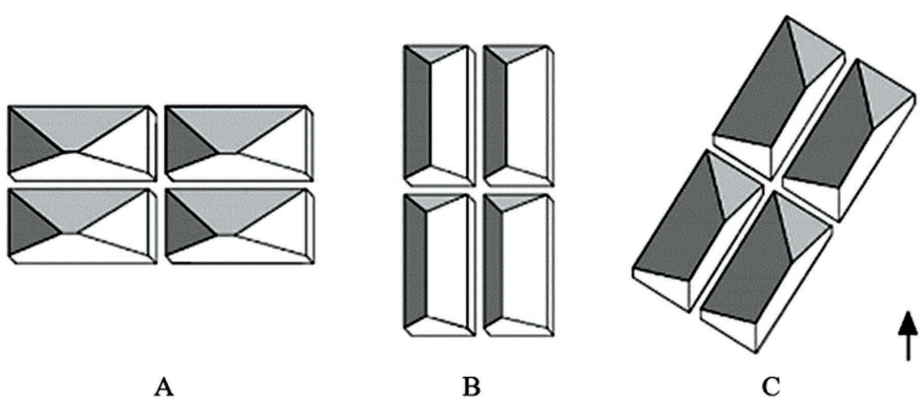
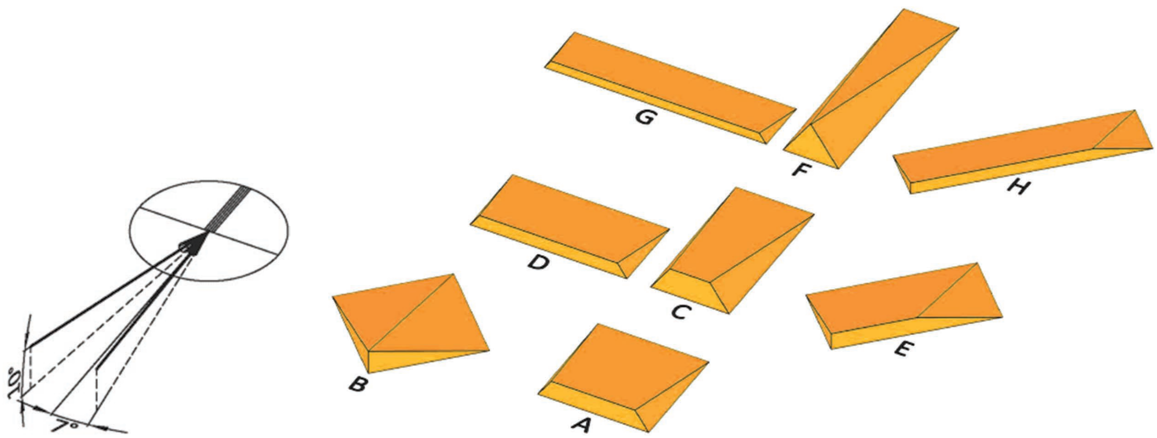


FIGURE 11. Solar envelopes for 1 hour of insolation in Aarhus (solar azimuth 7°, altitude 10°). Due to low sun, the south facing aspects of the volumes are quite small compared to Izmir (Figure 6). Only envelopes C and F have a substantial height and volume due to their long size towards north.



Volume Coefficient of case A for 1 hour of insolation was considered as the local measure in assessing the other coefficients. The results in Figure 12 appear similar to the corresponding ones in Izmir (Figure 9), indicating that latitude has no major effects on the *relative* size of Solar Volume Coefficients between different plots on the same location. However, the *absolute* values of the coefficients differ substantially between the two sites due to very low solar angles in Aarhus as shown in Table 2.

FIGURE 12. Solar Volume Coefficients in Aarhus, as percentage of the local reference case A, shown in blue at the bottom. For insolation duration of 1 and 2 hours, the average relative difference from Izmir values is negligible, but for 4 hours reaches -16%. In all cases, envelope F has the highest value, confirming the advantage of its plan proportions for northern latitudes too, as in Izmir.

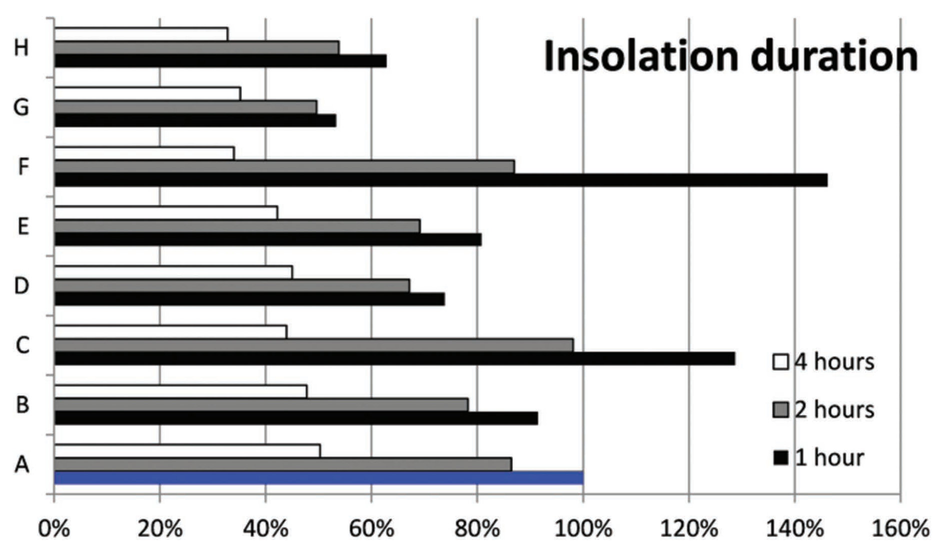


TABLE 2. Absolute values of Solar Volume Coefficient for 1, 2, 4 hours of insolation in Izmir & Aarhus in meters. In each duration length, the difference between the two locations is quite similar in all plot cases. The underlined numbers are the values used as relative measures in each location.

	1 hour			2 hours			4 hours		
	Izmir	**Aarhus**	*Ar / Iz*	**Izmir**	**Aarhus**	*Ar / Iz*	**Izmir**	**Aarhus**	*Ar / Iz*
A	**24.3**	**8.2**	*34%*	21.7	7.1	*33%*	14.4	4.1	*29%*
B	22.2	7.5	*34%*	19.6	6.4	*33%*	13.9	3.9	*28%*
C	30.9	10.6	*34%*	24.0	8.1	*34%*	12.2	3.6	*30%*
D	18.1	6.1	*34%*	17.0	5.5	*33%*	13.3	3.7	*28%*
E	19.6	6.7	*34%*	17.3	5.7	*33%*	12.3	3.5	*28%*
F	33.6	12.0	*36%*	20.8	7.2	*34%*	9.4	2.8	*30%*
G	13.1	4.4	*34%*	12.6	4.1	*32%*	10.5	2.9	*27%*
H	15.3	5.2	*34%*	13.5	4.4	*33%*	9.6	2.7	*28%*

The application of solar envelope on high latitudes requires low rise buildings, with relatively limited south-facing facades that do not benefit much from direct solar access. That undermines the objectives of the application, especially in regions where diffuse sky radiation competes direct in intensity, thus making solar applications on the roof—flat or pitched—rather more efficient than on south walls (Stasinopoulos 2002).

4.5 Effects of ground slope & orientation

Figure 14 shows that Solar Volume Coefficients on a south-facing ground slope in Izmir are higher than horizontal by 19% for every 10% of slope. This is because shadows are shorter due to the slope, allowing a taller envelope.

Figure 15 shows that if the orientation of the slope is reversed from south to north, then the solar envelope is reduced at the same rate as previously due to longer shadows. Because of such reduction, the application of solar envelope on north-facing areas enforces low buildings, as in the case of Aarhus.

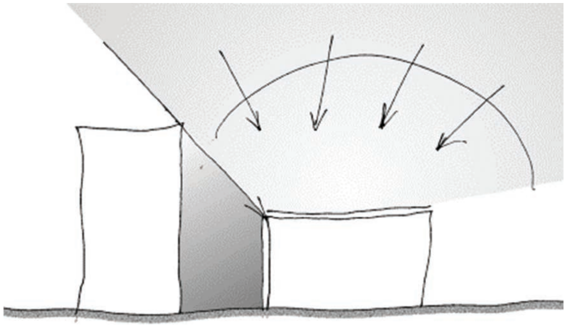


FIGURE 13. If solar access on south elevation is obstructed, the roof can offer a sizable alternative for solar applications in regions with increased amounts of diffuse radiation from overcast sky.

FIGURE 14. Effects of south-facing ground slope on Solar Volume Coefficient for 2 hours insolation as percentage of the reference case shown in Figure 9 in red (reference value is for 1 hour). The coefficients increase about 19% for every 10% of slope.

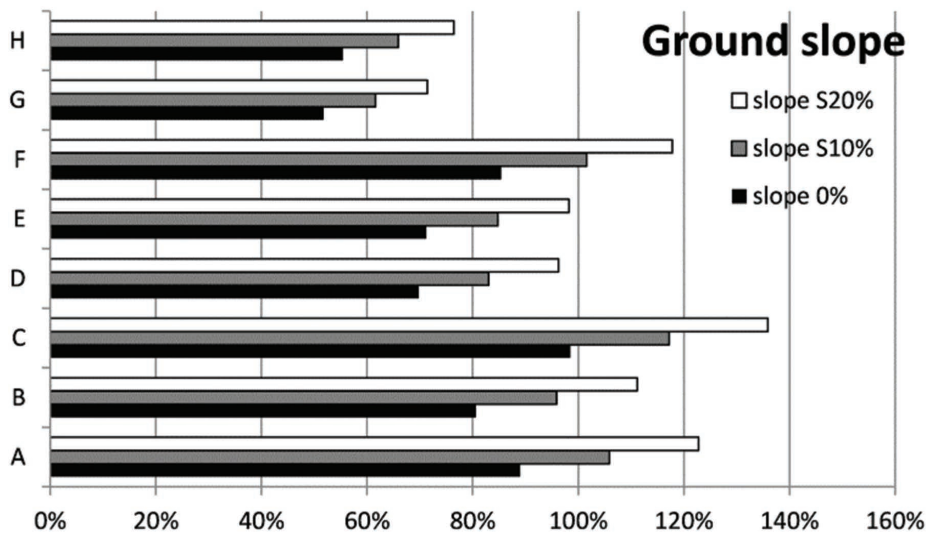
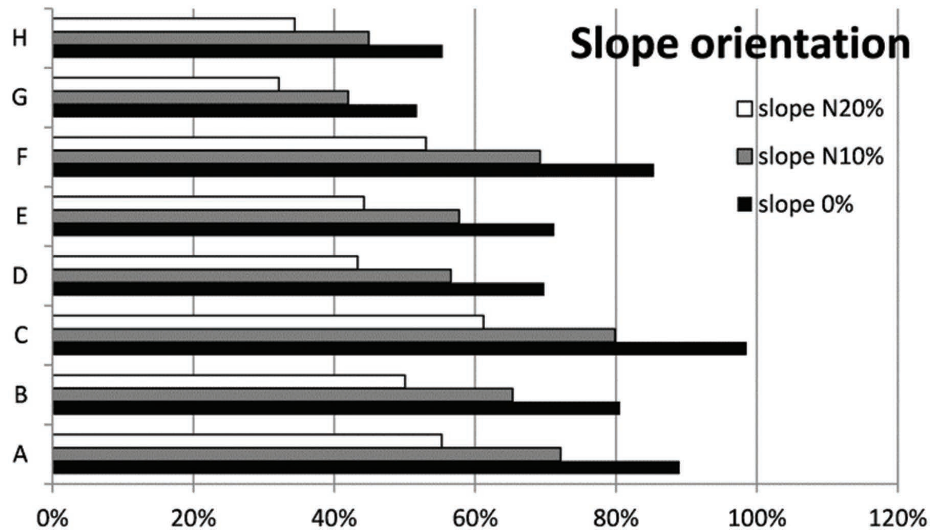


FIGURE 15. Effects of north-facing ground slope on Solar Volume Coefficient for 2 hours insolation as percentage of the reference case shown in Figure 9 in red (reference value is for 1 hour). Reversing the trend of south-facing slopes (Figure 14), the coefficients decrease about 19% for every 10% of north-facing slope. Further slope increase creates too low envelopes, as I happens even on horizontal ground at higher latitudes (e.g. Aarhus, Figure 11).



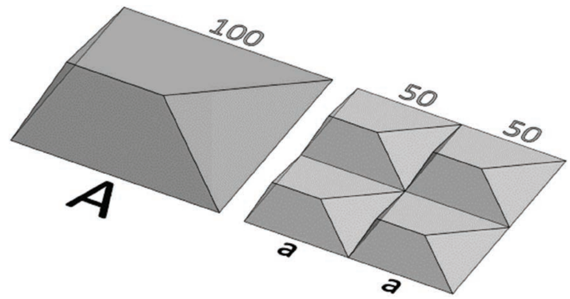


FIGURE 16. Effects of plot size: Solar envelopes on similar bases are similar solids. The volume proportion depends on the cube of the scale factor—e.g. the volume of envelope (A) is 8 times larger than of envelope (a). The Solar Volume Coefficient equals to the scale factor—e.g. it is double in (A) than in (a).

4.6 Effects of plot size

An important factor for the volume of a solar envelope is the size of its base. Small and large plots of the same shape and orientation have similar envelopes but of different volume, depending on the size of each plot. The ratio between the Solar Volume Coefficients of two similar cases equals to their scale factor (Figure 16). That underlines the fact that the application of solar envelope is more beneficial in terms of buildable volume for large land parcels than small ones.

5. URBAN SCALE APPLICATION

5.1 The objective: Assess 'Solar Building Potential'

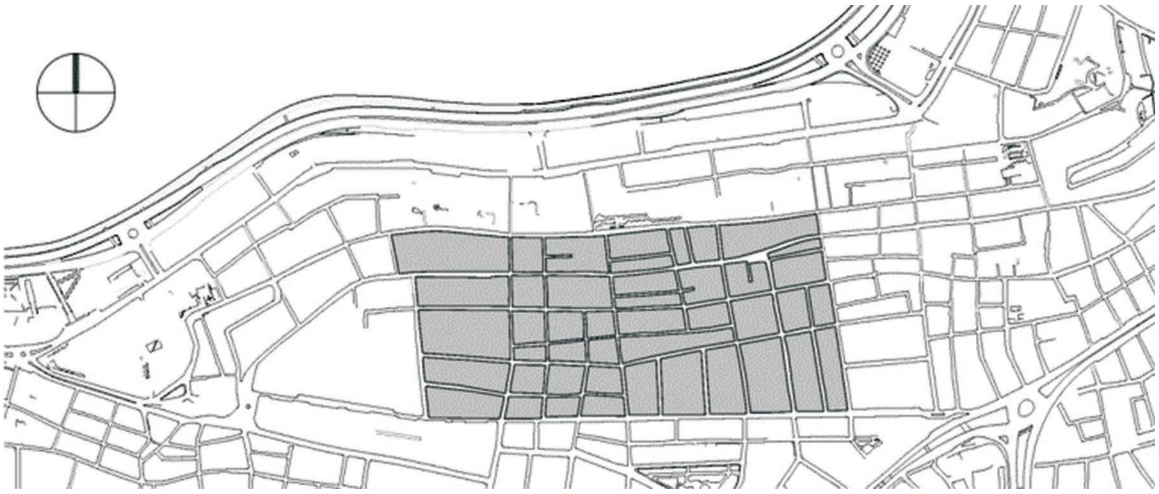
The same procedure that was used to compare single plots in previous chapter 4, is also applied in multiple plots simultaneously, to test the applicability in urban scale of the solar solid modelling method described in APPENDIX 1, as well as to examine the effects of ground slope and orientation on the *'Solar Building Potential'* of urban areas. The latter is a *quantitative* term, denoting here the total volume of solar envelopes over an urban area of a given street and block pattern. The volume determines the maximum size of buildings allowed to be built in the area, while facilitating a desired level of insolation on building facades of the neighbours. The Solar Building Potential is compared here to the maximum volume allowed to be built by current regulations, testing ways to bring the two values closer by modifying the solar envelope restrictions. The volumes of both, solar envelope and conventional urban prisms, are considered as fully built, without voids on the ground or above, indicating the maximum building volume (not floor area) that can fit inside each form independently of other restrictions.

An additional objective of this study part is a *qualitative* one: to visualize the *'solar cityscape'*, i.e. the results of solar envelope rules on the urban landscape. Although the solar envelope aims mainly in safeguarding solar access on south-facing facades, its effects on building forms provide more benefits like increased daylight onto street level, improved ventilation, or longer vistas between buildings since the narrow 'urban canyons' are replaced by milder 'urban valleys'. In addition, they offer a more diverse urban geometry than vertical prisms, enhancing the visual variety of urban environment.

5.2 Testing context

The testing ground of this study part is the same location mentioned above in 3.3: A dense area of about 13 hectares in the Mediterranean city of Izmir (latitude 38.4 N, longitude 27.1 E), with 44 irregular blocks, each taken as a single land parcel with a maximum building height of 15m set by the current local regulations. The required insolation period is taken between 11–13:00 solar time on December 21. The particular area was not selected as a candidate for

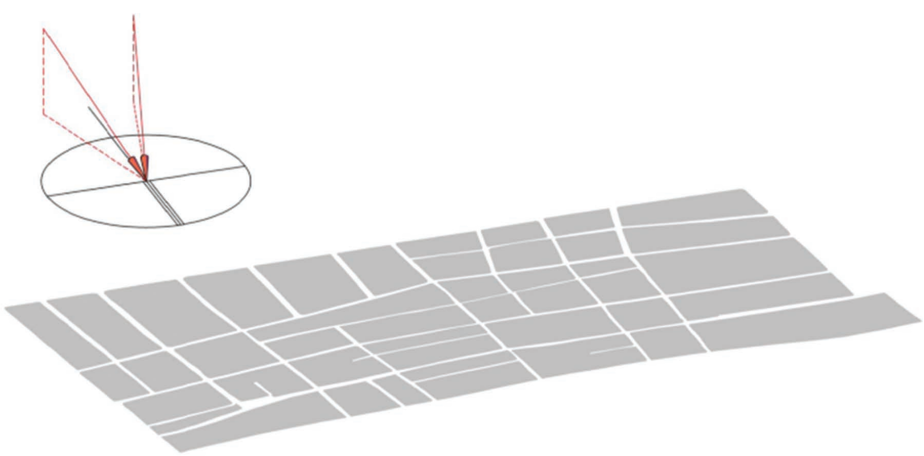
FIGURE 17. The street pattern of a dense urban area of Izmir is used for testing various factors affecting the size of solar envelope.



developing a real solar envelope scheme, but only as a random example of street pattern that is typical in many similar urban areas of the wider region.

The ground of the area is first considered horizontal, applying the process illustrated in Figure 19 to Figure 22. Then all solar envelopes are re-constructed for a 10% slope towards N-NW (actual), and again for 20% (hypothetical), in order to check the effects of ground slope on the total volume of solar envelopes—i.e. the Solar Building Potential of the area. The process is repeated assuming the reverse north direction, to check the effects of orientation on that potential.

FIGURE 18. Axonometric view from NE of the same blocks marked in Figure 17 with their surroundings removed. The cut-off period is taken between 11–13:00 solar time on December 21, shown by the solar vectors.



5.3 Multiple solar envelope construction

Each urban block of the selected area is considered like the individual plots of Figure 5, applying the same solid modelling technique as in APPENDIX 1 but in larger scale and with a height restriction of 15m. The final result is the solar cityscape shown in Figure 22, totally different than the current one (Figure 19).

FIGURE 19. The area fully covered by 15m high blocks, showing the maximum allowable building volume. Voids within the blocks are not shown.

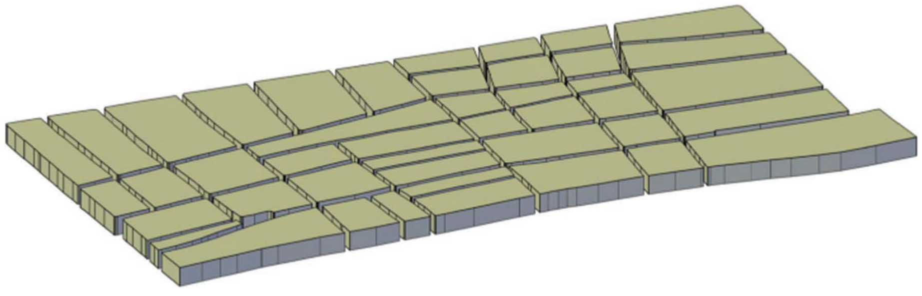


FIGURE 20. The morning solids of all blocks, as in Figure 28. The solar vector is much longer than needed to ensure sufficient height of the constructed solids.

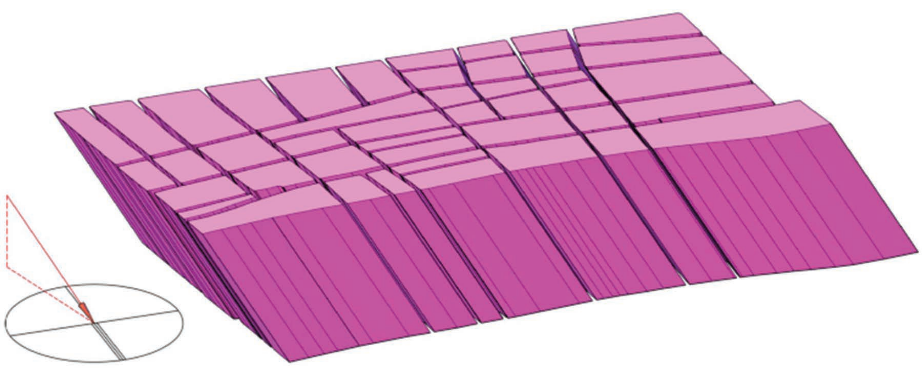


FIGURE 21. The afternoon solids, as in Figure 29.

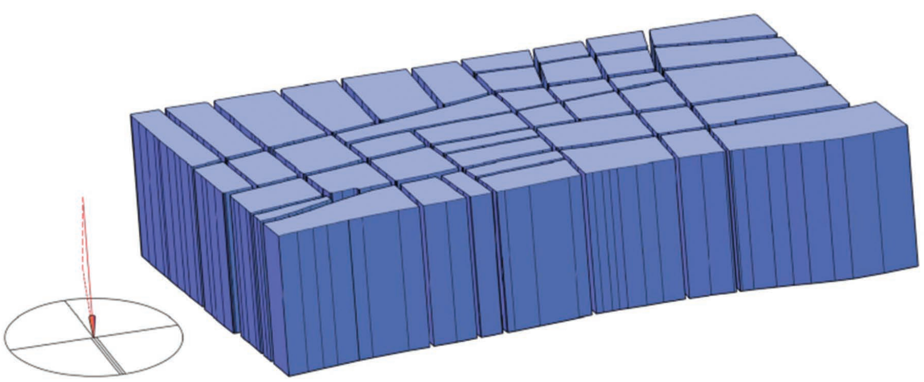
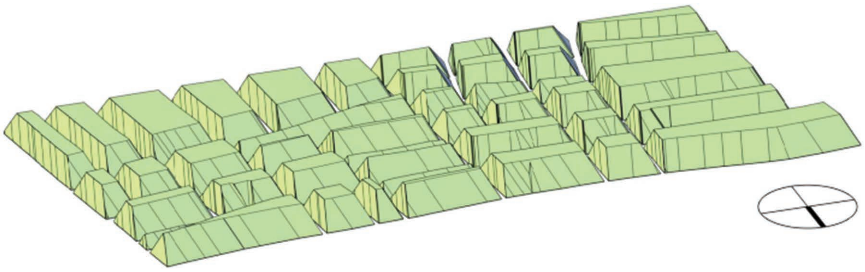


FIGURE 22. The intersections of the three solids on each block produce the solar envelopes over the blocks of the area. The Boolean extrusions in the previous steps are performed once for all blocks together, but the intersections require individual handling of each block. That is the only time-consuming part of the whole modelling process.



5.4 Adjusting solar envelope volume

The height restrictions imposed indirectly by the solar envelope can reduce substantially the volume allowable for a building scheme compared to usual planning restrictions. For instance, the volume of the solar envelope in Figure 22 is 60% of the conventional volume in Figure 19.

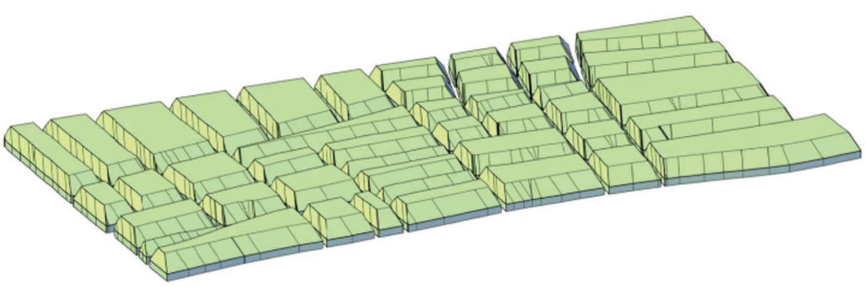
To mitigate the undesirable decrease on the use of precious land because of such limitations, a solution is to raise the reference base of the solar envelope (*'solar fence'*). That increases the available volume on the bottom part of the building, which in dense urban areas is usually occupied by commercial functions that do not benefit from solar access. This was applied on the Izmir example by raising the base of the envelope by adding a *'plinth'* of 6m and trimming any tops that exceeded the height limit due to the rise (Figure 23). As a result, the available total volume increased to 85% of the conventional one shown in Figure 19.

Besides the solar fence, there are other factors that affect the volume of the solar envelope. The following paragraphs summarize an experimental survey on the effects of such factors on the Solar Building Potential of the area: ground slope, orientation, and maximum building height, with and without the use of solar fence (referred as 'plinth'). The various quantitative results are presented in Figure 24. The axonometric views in APPENDIX 2 visualize the solar cityscape created in each case, for comparisons with the conventional one shown in Figure 19.

Effects of ground slope

The selected area is not actually horizontal as assumed in the previous paragraphs, but it has a 10% slope towards N-NW. The previous procedure is applied again for that ground slope,

FIGURE 23. Solar envelopes are raised by 6m to increase building density. Tops are trimmed to observe restrictions for maximum height at 15m.



resulting to the cityscape shown in Figure 33 without and with plinth. Similarly, a new application on a hypothetical slope of 20% gave the cityscape of Figure 34.

Effects of ground slope orientation

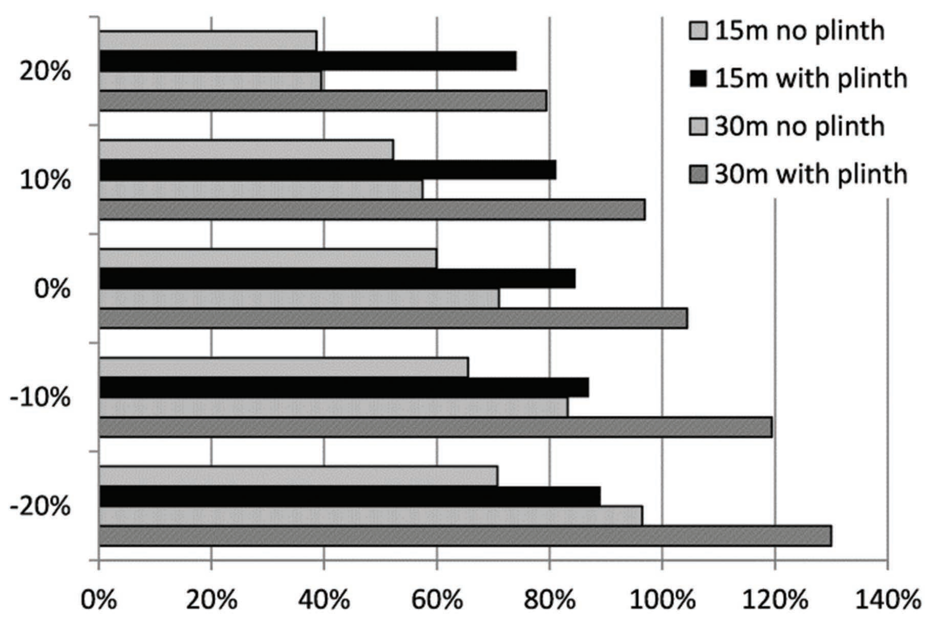
To test the effects of different slope orientation, the same procedure as previously was repeated assuming the reverse orientation on ground slopes of 10% and 20%, generating the cityscapes shown in Figure 35 and Figure 36. Due to the south-facing slope of this layout, the solar rays are at higher angle from the ground, thus generating solar envelopes with larger volume than on a north-facing slope (Figure 24).

Effects of height increase

The previous applications were limited by a maximum allowable height of 15m, which is imposed—among other reasons—in order to amend the daylight reduction caused by the restricted sky view in the urban canyons. Since the solar envelope by definition promotes solar access instead of obstructing it, it can extend much higher without causing daylight deficits. Here a height of 30m was taken as maximum to test the effect of height on Solar Building Potential (a maximum height value already applied in other parts of the city).

Figure 24 shows that doubling the maximum allowable height from 15 to 30 meters affects the volume of envelopes not proportionally to the doubling of height. The volume increase appears mainly on south-facing slopes, while the difference is low on north-facing ones. In contrast, adding a plinth has more effect on envelopes situated on north- rather than south-facing slopes. It is noteworthy that a combination of increased height and raised solar fence (plinth) can provide more buildable volume than the current dense prisms practice, offering the additional benefits of urban 'valleys' instead of 'canyons'.

FIGURE 24. Comparison of Solar Building Potential on ground of various slopes and orientation as percentage of the maximum buildable volume without solar envelope rules (100%), for 2 hours of insolation in Izmir. Y-axis refers to ground slope (positive slope means north facing, negative south). Maximum height is 15 and 30m.



6. DISCUSSION

6.1 Competing densities

Figure 24 shows that the Solar Building Potential is generally inferior to the building density that is provided by the typical prismatic urban block. That is occasionally an argument against the implementation of solar-based restrictions in planning rules. However, it is possible to match that density under certain favourable conditions, either *natural* (slope, orientation) or *regulatory* (solar fence, maximum height).

The use of solar fence is quite appropriate for commercial areas, where not many solar applications are usually found at low level. In the present case study, such 'plinth' of 6m adds about 40% to the volume of plain solar envelope on horizontal ground. In the most and least favourable slope and orientation (20% facing south or north) the additional volume due to raised solar fence is found as 92% and 26% respectively.

The increase of maximum allowed height is another plausible technique to increase Solar Building Potential. One of the reasons of enforcing height limitations has been the provision of daylight into the urban canyons. But the solar envelope, given its geometric properties, does not obstruct solar access, independently of its height. Therefore high rise buildings can be accepted in urban design under solar envelope rules without causing the common problem of overshadowing.

As said, in addition to the energy benefits, the solar envelope improves the overall conditions of the urban environment in terms of light, ventilation, and views, thus compensating *qualitatively* for any *quantitative* reduction of building volumes.

6.2 Plot sizes

The observations on the randomly selected location are based on the existing pattern of urban blocks. As shown in chapter 4, Solar Volume Coefficients differ between single plots according to parameters like their proportions and orientation. Therefore the Solar Building Potential of the examined area could be increased by rearranging its plots in a more favourable layout according to the desired duration of solar exposure. In practice, that means to increase the number of plots oriented along N-S direction, as well as the square ones (cases C, A, F in Figure 7 ordered according to data in Figure 9).

As illustrated in Figure 16, small land parcels cannot yield a high Solar Building Potential compared to similar but larger ones under the same conditions. Therefore, in any case, the size of the plot is the most influential factor on the volume of its solar envelope. In the examined location, each urban block was considered as a single plot, although in reality it includes several individual properties. That observation indicates that, in order to avoid low densities, the application of solar envelope in urban planning should consider urban blocks as one entity instead of separate smaller ones based on individual properties. That is similar to the enforcement of, say, building lines that apply to the entire block front, independently of property divisions.

The problem of solar penetration to the interior of large volumes is an architectural problem that can be solved by removing part-s of the solar envelope volume through the introduction of atria or other voids that reduce the depth behind the facades. This approach is similar to the inner courtyards in standard urban blocks (Figure 25).

6.3 Solar envelope applicability

The exemplary cases here indicate that the solar envelope cannot be universally beneficial in terms of Solar Building Potential. Apart from the plot size, the ground slope and its orientation

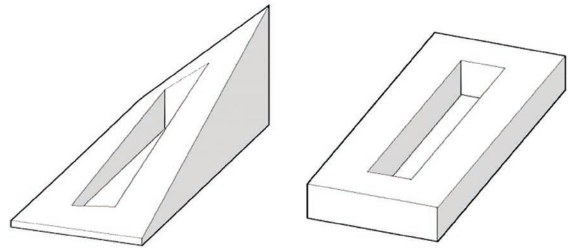


FIGURE 25. An example of a large pyramidal volume with an inner cut to admit sunlight to the interior. Similar to VIA 57 West residential block in New York, by Bjarke Ingels Group (BIG). The inner courtyard can be applied in a pyramidal form like in a common prismatic urban block.

is a factor that can lead to very low Solar Building Potential—even 'negative' in cases of north-facing terrains at a slope angle exceeding the solar altitude during the cut-off period. Similarly, the feasibility of solar envelope on northern latitudes with low sun paths is questionable.

However, solar envelope geometric principles can be utilized as a model for relating the urban fabric with the dominating element of sun and its rhythms, not so much aiming to support solar applications, but to ameliorate our cities that increasingly suffer from high densities and the associated problems of reduced daylight, poor ventilation, flora-hostile conditions, or limited sky-views.

The solar cityscapes illustrated in Figure 22, Figure 23, and APPENDIX 2 indicate that the urban fabric can acquire a rich volumetric variety not based on individual whims or random circumstances, but regulated by a nature-guided order.

6.4 The tools

The numerous solar envelopes presented here were created very fast through the use of solid modelling tools, mainly using commands for extrusions along solar vectors ('EXTRUDE' in AutoCAD) and Boolean intersections ('INTERSECT'). Another indispensable tool for speedy work was the command for finding the volume of any complex solid form ('MASSPROP'). Without such computer means, it would be impossible to deal with the geometrical complexity of solar envelopes and their diverse parameters.

7. CONCLUSIONS

The present work started as a test of the procedure described in APPENDIX 1 in order to evaluate the simplicity and speed of its use. The first experimental applications showed that solid modelling can provide a powerful tool in planning for solar access, through a simple process characterised by high speed and accuracy. The method can be applied to a variety of plot circumstances: horizontal or inclined, coplanar or not, single or in groups.

With solid modelling as a powerful tool, the research expanded into a widespread survey on the numerous variations of solar envelopes according to the parameters affecting its shape and size: (a) plot size, proportions, and orientation; (b) terrain slope and orientation; (c) required insolation duration; (d) maximum height and solar fence; (e) latitude. The combinations of these parameters create diverse forms and volumes of solar envelopes, as demonstrated by numerous examples.

The study focused first on single plots of selected attributes, to calculate their Solar Volume Coefficients as a measure of comparison between the various cases. Then it went on a large urban area to calculate the Solar Building Potential of many plots together. The main reference location was (38°N) Izmir, and briefly Aarhus (56°N) for a comparison between latitudes.

The multiple calculations showed the influence of the factors involved in the geometry of solar envelopes. They also showed that solar envelopes generate urban densities lower than conventional regulations on urban solid geometry. However, given certain conditions, that can be altered if need be, as shown by the study.

In addition to facilitating solar applications, the introduction of solar envelopes in urban planning can enhance the urban environment in terms of daylight, ventilation, and vistas. The visualization of solar cityscape consisting of many solar envelopes together, shows that the sun can have a fertile influence in shaping and upgrading our cities not only in practical matters but visually too. The presented tools and procedure can contribute to that direction.

8. APPENDIX 1

Solar solid modelling

The construction of a standard solar envelope requires three geometric items as entry data: The site boundary, maximum height restrictions, and the solar angles at the selected cut-off time. Starting from that, the envelope can be constructed as a 3D surface using standard CAD commands to create solids through extrusion and to apply Boolean operations on them. Figure 26 to Figure 31 exemplify such an application using AutoCAD 2017 on a simple imaginary plot, following the steps originally described by Stasinopoulos (2000). The same procedure was followed throughout this work. The size of each solar envelope volume was computed using the command MASSPROP of AutoCAD.

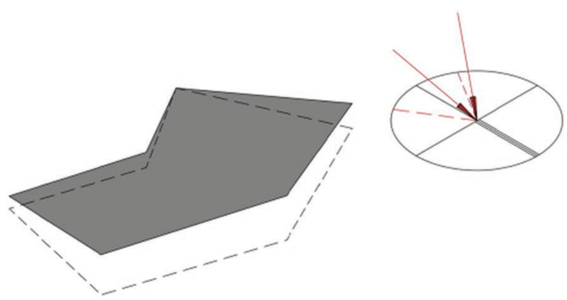


FIGURE 26. A simple plot with a concave boundary on a slope is used to exemplify the method. Orientation is indicated by the circular compass. The vectors show the directions of solar rays at the ends of the specified cut-off time on winter solstice, before and after solar noon ('morning' and 'afternoon' solar vectors). The solar angles can be found by commands available in the modeller, by online solar calculators, or by a sun path diagram. Dotted lines are projections on horizontal.

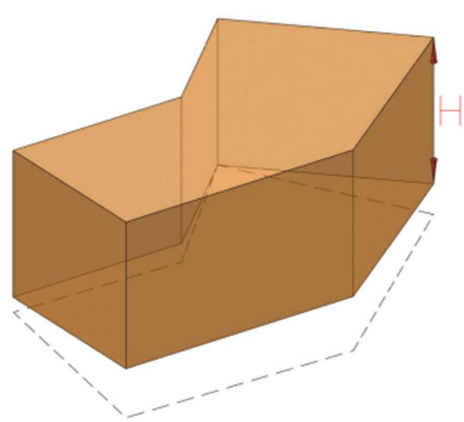


FIGURE 27. The maximum allowable volume on the plot is found by extruding the boundary vertically at a distance equal to the maximum permitted height H.

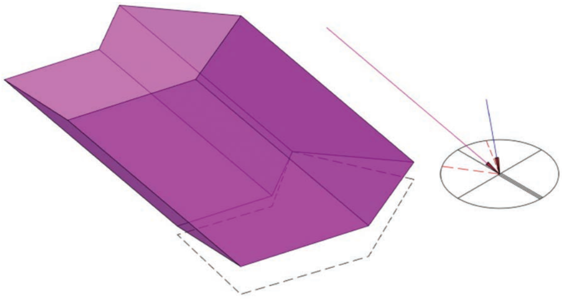


FIGURE 28. The boundary is extruded along the morning solar vector at a length sufficiently longer than height H. The created 'morning solid' does not cast any shadow outside its base when the specified insolation period starts.

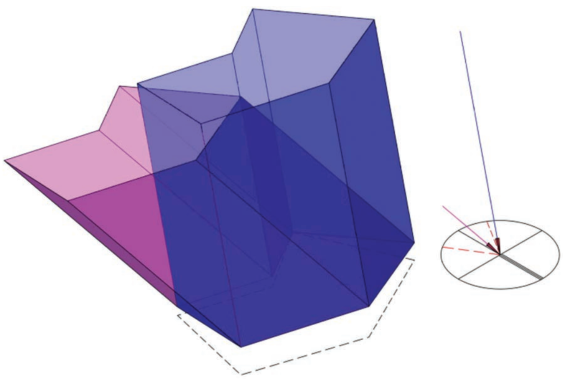


FIGURE 29. The extrusion is repeated for the afternoon solar vector. Similarly, the 'afternoon solid' does not cast shadows outside the plot at the end of the insolation time.

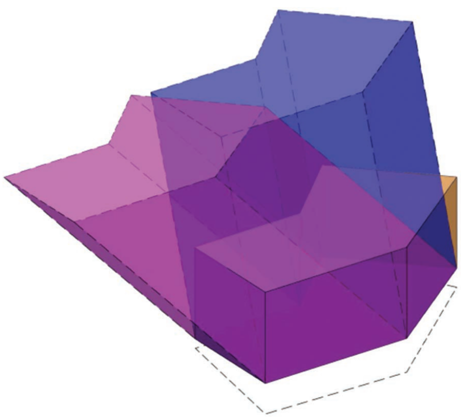


FIGURE 30. The solar envelope is the intersection of 3 solids: The maximum buildable volume (Figure 27), the morning solid (Figure 28), and the afternoon solid (Figure 29).

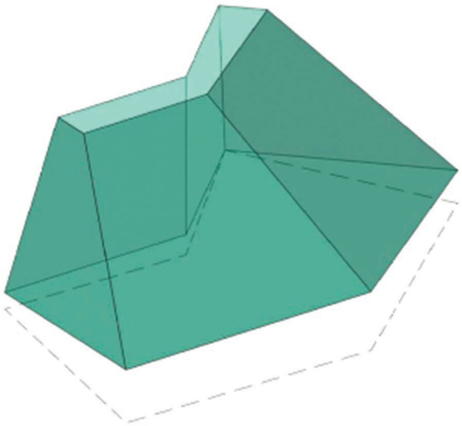
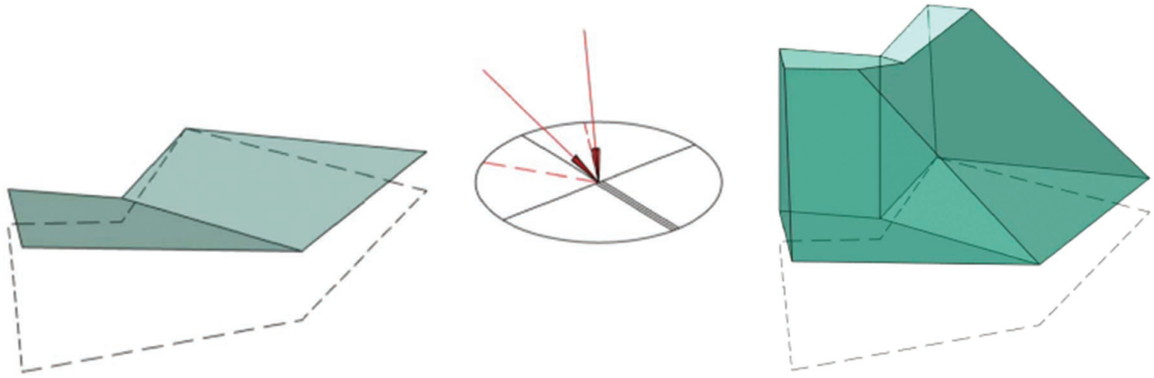


FIGURE 31. The entire surface of the created solid is lit by the sun during the insolation period and casts no shadows. As the sun moves from 'morning' to 'afternoon' position, the projection of any point of the solar envelope onto the ground remains within the base, moving towards the south side and closer to the middle of the plot, without crossing the boundary.

FIGURE 32. The plot shown here has the same horizontal contour as before, but consists of two non-coplanar parts. The solar envelope is constructed following a similar procedure as for coplanar plots.



If the boundary of the plot is not coplanar, then the plot is divided into coplanar parts. The same steps as previously are applied on each part separately, then the solids created at each stage are united by a Boolean operation. The final Boolean intersection is applied on three solids, maximum, morning, afternoon, as in the single coplanar plot (Figure 32).

9. APPENDIX 2

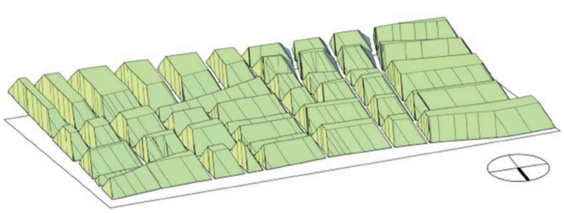
Solar cityscape variations

The images below present various types of solar cityscape, generated on the same area of Izmir under different assumed values of ground slope, orientation, and maximum allowable height. Each case is shown without and with a solar fence of 6m ('plinth'). The images illustrate the bounding volumes of buildings, not the buildings themselves. Quantitative data on Solar Building Potential of the various configurations are shown in Figure 24.

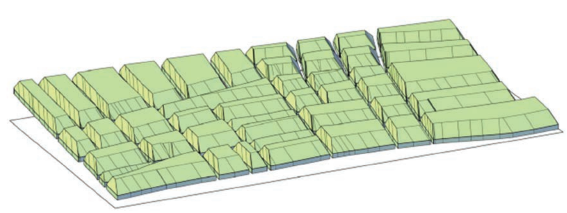
Effects of ground slope (H_{max} = 15m)

Orientation and maximum height remain the same as previously (Figure 22 & Figure 23), but the ground has a north facing slope of 10% and 20%.

FIGURE 33. Solar cityscape on north facing 10% slope.

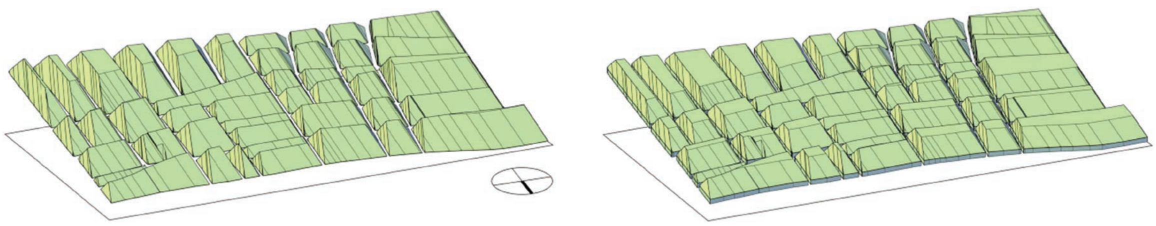


No plinth



With 6m plinth

FIGURE 34. Solar cityscape on north facing 20% slope.

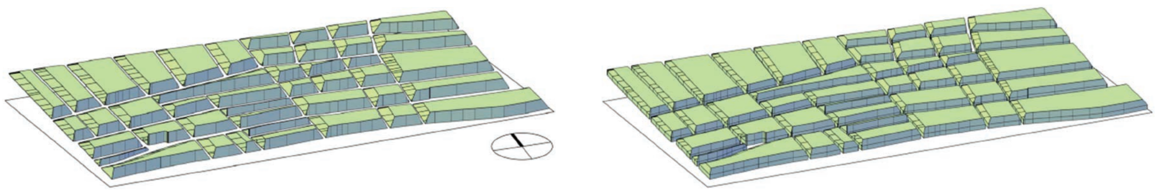


No plinth With 6m plinth

Effects of ground slope orientation

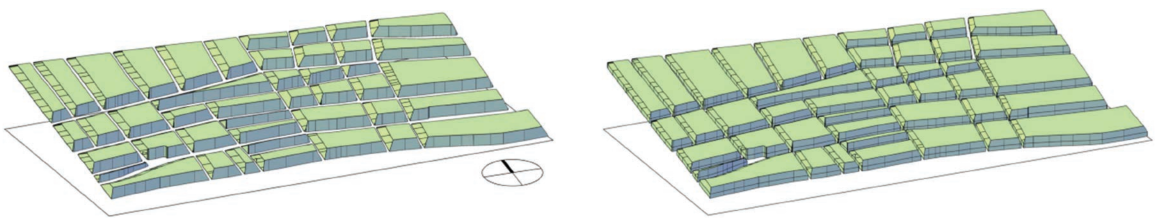
Slope orientation here has changed from north to south facing.

FIGURE 35. Solar cityscape on south facing 10% slope.



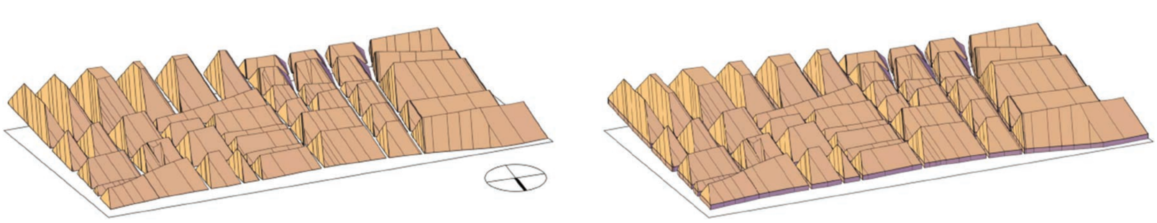
No plinth With 6m plinth

FIGURE 36. Solar cityscape on south facing 20% slope.



No plinth With 6m plinth

FIGURE 37. Solar cityscape on north facing 10% slope (H_{max} = 30m).

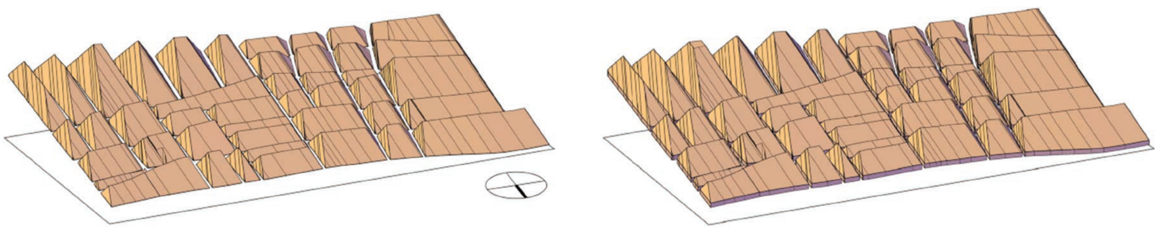


No plinth With 6m plinth

Effects of height increase

The same settings are used here, except maximum height that is increased from 15 to 30m.

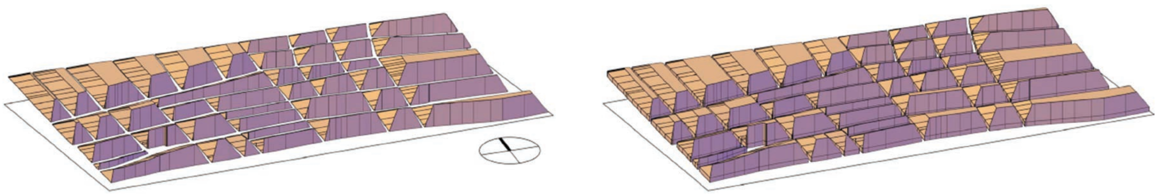
FIGURE 38. Solar cityscape on north facing 20% slope (H_{max} = 30m).



No plinth

With 6m plinth

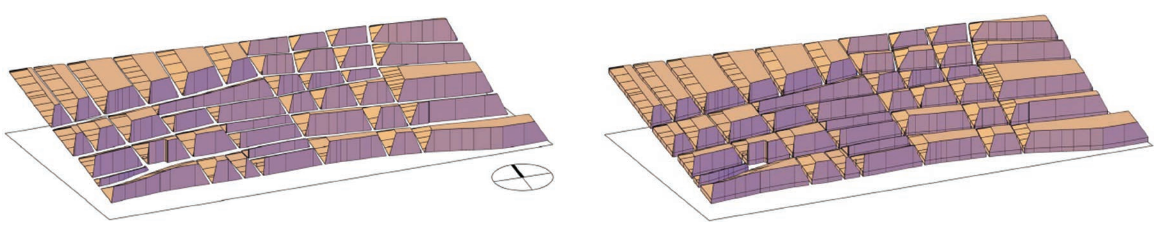
FIGURE 39. Solar cityscape on south facing 10% slope (H_{max} = 30m).



No plinth

With 6m plinth

FIGURE 40. Solar cityscape on south facing 20% slope (H_{max} = 30m).



No plinth

With 6m plinth

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